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## Design and energy optimisation of a hybrid flywheel bus rapid transit

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**Abstract:** A hybrid flywheel powertrain system for a bus rapid transit (BRT) is designed in this paper, using a technical-economic study. The modern high-speed flywheel, which is used in the modern heavy vehicle hybrid powertrain, has high efficiency, low weight, and low initial cost. In contrary to the other powertrain hybridisation options, the energy conversion does not need using the flywheel energy storage system. Therefore, the power charge and discharge process are high-speed and efficient. The economic study is defined based on component price and fuel consumption cost. The model of the conventional and the hybrid bus is generated in AVL CRUISE software. Results show that the hybrid flywheel bus has better efficiency in BRT driving cycle in comparison with the normal Tehran City bus driving cycle. The optimal flywheel capacity is 1.4 MJ to use in Tehran BRT, which has ten months payback period. The reduction in fuel consumption is 28.61%.

**Keywords:** bus rapid transit; BRT; hybrid flywheel powertrain; energy conversion; AVL CRUISE software; technical-economic study.

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## 1 Introduction

In recent years, researchers investigate many solutions to improve energy efficiency for both vehicle and vehicle components. As we know, for achieving a high level of energy efficiency, we should improve the powertrain of vehicles. Besides that, in order to improve vehicle energy efficiency, we consider energy storage systems (Kobayashi et al., 2009). One of the moving vehicle from the viewpoint of energy saving was hybrid vehicles (Ahmadizadeh and Mashadi, 2016) which could recover vehicle kinetic energy that might otherwise be lost as heat during braking. One great idea that flywheel proved the form of energy storage on account of its energy density and high power, broad operating, useful lifetime, high efficiency, temperature variety, and independence from depth-of-discharge effects (Amodeo et al., 2009; Liu and Jiang, 2007). Electric hybrid vehicles waste close to 64% of the braking vitality that might potentially be recuperated on the move forward those fuel economy (Boretta, 2010). Those perfect gas results will be on keep away from transformations starting with particular case structure from claiming vitality to another by keeping those vehicles vitality in the same form, similarly as at those vehicles begins braking at the vehicle is moving down with speed (Mishina and Muromachi, 2017). That flywheel gives a perfect type from claiming vitality cradle is stockpiling because on its excellent power denseness and cycle term (Jefferson and Ackerman, 1996).

In Formula 1 racing cars, the flywheel system has been utilised as short-term kinetic energy storage since the instructions were modified in 2009, allowing such equipment. The supplier of this flywheel energy storage system, which was used in Formula 1, was the company hybrid systems (Original F1 System, 2016). Cross and Brockbank proposes a mechanical hybrid system for the US-FTP75 cycle. It has been shown that kinetic energy stored during braking and regenerated to propel the vehicle can supply about 21% of the energy required to drive the complete cycle (Cross and Brockbank, 2009). Li in 2015 shows that his strategy for regenerative braking can ensure vehicle safety during emergency braking situations and improve the recovery energy up to 17% (Li et al., 2016).

Some new powertrain options are studied for city buses in recent years. Sánchez in 2013 shows that the battery-electric bus has 25.62% better efficiency in comparison with a conventional bus. Zhang in 2013 study and test on braking energy regeneration control of an electric bus. His test results show that the application of the lithium battery is advantageous over the Ni-MH battery, improving the hydrogen consumption by 11.5%. The hybrid flywheel powertrain has better performance in comparison with the other new powertrain options (Iwata and Matsumoto, 2016). Xiong in 2008, uses two fuzzy logic

modules in the energy management system of an electric city bus. He shows that the energy consumption is theoretically reduced by 30.3% to that of the conventional diesel bus while the battery state-of-charge is kept in a healthy range (Xiong et al., 2009). Tianjin's Government had placed about 450 electric city buses into working in 2012 and will raise the total quantity to 2000 by 2015 (Peng et al., 2015).

The first two hybrid flywheel buses were publicised in Switzerland in 1950, and they were working for ten years. The concept was carried to Congo, where 12 flywheel hybrid buses were working. After that, in 1956, Belgium was the next country to take on the gyro-bus. In the 1960s, all of the buses were gone from the bus fleet (Hayes et al., 1999). In the 1990s, at the University of Austin, Texas, a bus was equipped with a high-speed flywheel. The energy capacity of this flywheel was 2 kWh, and its power was 150 kW of continuous power (Flynn et al., 2005). Delhi in 1992 was modelled and simulated the flywheel energy storage system for a city bus. It is achieving 35.5% fuel saving in the simulation (Tripathy, 1992). In 2005, Fraunhofer developed an articulated 18 m long bus that can be powered by the flywheel energy storage system (McGroarty et al., 2005). In London, there will soon be 500 buses out on the streets, equipped with a 1.44 MJ flywheel, enough to accelerate the bus from 0~50 km/h and regeneration the energy when braking. This system is saving about 20%~25% of the energy (Hedlund et al., 2015).

Regarding previous studies, there are few specific types of research on modelling of hybrid engine-based buses with the flywheel as a secondary storage system. Hence, this paper aims to show the results of modelling a hybrid flywheel bus to use in the different driving cycles and shows the benefit of using hybrid flywheel configuration to use in bus rapid transit (BRT). In this paper, an economic optimisation problem is proposed to determine the capacity of the flywheel to operating in the BRT driving cycle (Zahabi et al., 2014).

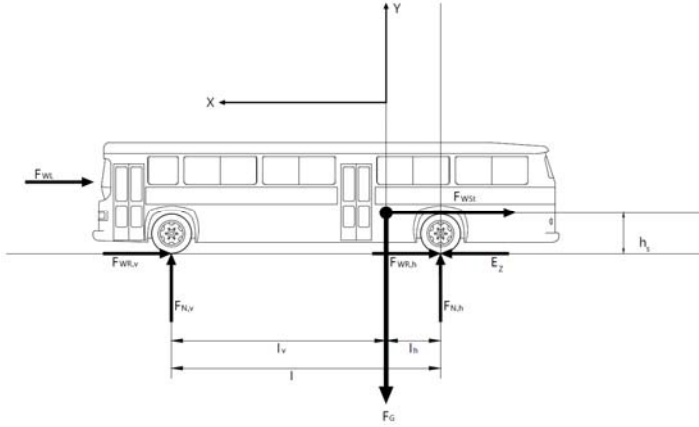
In the next section, the mathematical modelling of the bus and the flywheel are presented and discuss the limitation of flywheel size. In that section, the flywheel specifications, such as maximum power and speed, are introduced. After that, the coupling configuration of the flywheel and controller is specified. At the end of Section 2, the simulation procedure of the hybrid flywheel powertrain in AVL CRUISE software is discussed. In Section 3, the flywheel sizing process is done. This sizing process is started by the initial design, such as the maximum speed, grade-ability, and acceleration tests. The flywheel cost is used for optimising energy capacity in the BRT driving cycle by calculating the payback period of the initial adding cost of the flywheel.

## **2 Modelling**

In this section, governing equations of bus and flywheel are introduced.

### *2.1 Bus equation*

To calculating the amount of energy required, a model of longitudinal dynamics is examined. In Figure 1, the bus model under the applied forces is shown.

**Figure 1** Buses in Tehran under the applied forces


To moving the bus, the driving force ( $F_W$ ) must overcome the resisting forces ( $F_Z$ ).

$$F_W > F_Z$$

$$F_W = F_{WR} + F_{WL} + F_{WB} \quad (1)$$

In this equation,  $F_{WR}$  resistance tires,  $F_{WL}$  air resistance, and  $F_{WB}$  acceleration resistance. By using these governing equations for driving and resisting forces, the maximum acceleration, top speed, and fuel consumption can be calculated. By the expanding equation (1) can calculate equation (2). Equation (2) show the amount of force necessary ( $F_W$ ) to overcome the resistance forces. The parameters of this equation involve rolling resistance forces  $F_R$ , air resistance  $F_{WB}$ , acceleration equals force  $F_{WB}$ , resistance due to tire slippage ( $F_{An}$ ), power transmission  $F_{Tr}$ , force demand to activate the power  $F_{Anci}$ .

$$F_w = F_R + F_{WB} + F_{WB} + F_{An} + F_{Tr} + F_{Anci} \quad (2)$$

The amount necessary to overcome rolling resistance is calculating by equation (3).

$$F_R = mgf_r v \quad (3)$$

In this equation  $m$  is vehicle mass,  $g$  is an acceleration of gravity,  $f_r$  rolling resistance coefficient, and  $v$  is vehicle velocity.

## 2.2 Flywheel equation

The energy stored in the flywheel is obtained from equation (4).

$$E_{FW} = \frac{1}{2} J_{FW} \omega_{FW}^2 \quad (4)$$

In this equation,  $E_{FW}$  is flywheel energy,  $J_{FW}$  is inertia of the flywheel and  $\omega_{FW}$  is the rotational speed of the flywheel. Flywheel's ability to withstand the centrifugal force is signified by the maximum rotational speed of the flywheel. Equation (5) determines the maximum energy stored in the flywheel that put the flywheel on the ultimate failure (Piróg et al., 2010).

$$E_{\max} = \frac{1}{2} m_{FW} \frac{\sigma'_{\max}}{\rho} \tag{5}$$

In this equation,  $E_{\max}$  is maximum flywheel energy,  $m_{FW}$  is mass flywheel,  $\sigma'_{\max}$  is the maximum tensile stress and  $\rho$  is the density of the material that was used in the flywheel. Equation (6) is the most general equation for the expression of flywheel energy.

$$\frac{E_{FW}}{m_{FW}} = k \frac{\sigma'_{\max}}{\rho} \tag{6}$$

In this equation  $k$  is a figure factor, figure factor is determined according to the flywheel so that the stress distribution of flywheel be the appropriate range. In Figure 2, the figure factor for various geometric shapes is displayed.

The maximum useful energy that the flywheel can deliver calls to the flywheel energy storage capacity. Flywheel energy capacity depends on the maximum and minimum operating flywheel speed. Equation (7) has been shown how to calculate the flywheel energy capacity.

$$E_{cap} = \frac{1}{2} J_{FW} (\omega_{FW,max}^2 - \omega_{FW,min}^2) = \frac{1}{2} J_{FW} \omega_{FW,max}^2 \left( 1 - \left( \frac{\omega_{FW,min}}{\omega_{FW,max}} \right)^2 \right) \tag{7}$$

In this equation,  $E_{cap}$  flywheel energy storage capacity,  $\omega_{FW,min}$  the minimum speed of the flywheel and  $\omega_{FW,max}$  is the maximum speed of the flywheel. Usually, the minimum flywheel speed is considered half of the maximum flywheel speed (Kaftanoglu, 2009).

**Figure 2** Figure factor for various geometric shapes

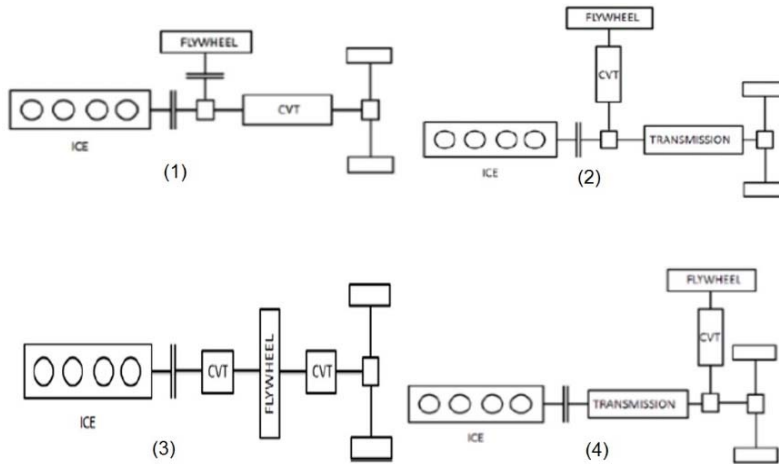
Flywheel geometry	Cross-sectional or pictorial view	Shape factor, $k_s$
constant-stress disc (OD $\rightarrow \infty$ )		1.000
modified constant-stress disc (typical)		0.931
truncated conical disc (typical)		0.806
flat unpierced disc		0.606
thin rim [(ID)/(OD) $\rightarrow$ 10]		0.500
shaped bar (OD $\rightarrow \infty$ )		0.500
rim with web (typical)		0.400
single filament bar		0.333
flat pierced disc		0.305

Source: Kaftanoglu (2009)

### 2.3 Coupling the flywheel system

One of the critical issues in the hybrid power transmission system is the location of the flywheel in the powertrain system. Figure 3 is displayed the flywheel status in the transmission.

**Figure 3** Flywheel status in the transmission



Source: Dhand and Pullen (2013)

In Figure 3, models (1), (2) and (4) are a parallel method, the combustion engine and flywheel are connected to a transmission system in parallel. Model (3) is used as a series method. In the combustion engine series, it does not directly connect to the power transmission system, but the combustion engine itself will charge the flywheel, and the flywheel will provide the power required to drive the car. In this method, it is not possible to use a two-stroke force in parallel, but both in the form of a single-stroke actuator. Direct to the system.

In model (1), engine and flywheel either directly to the transmission where the continuously variable transmission (CVT) is connected. Both flywheel and engines have a separate clutch, if necessary, any of them can be removed from the power transmission system. In this method, both systems are coupled together and provide the required power transmission system. In this control system, the power split between two power generators plays an important rule because, at each moment, the control system must calculate the required power level and determine the optimal power of each power generator. The advantage of this system is that the engine can operate at their optimum; the disadvantages of this system require sophisticated control (such as developed in Safaei et al., 2015) and ability coupled flywheel and engine speeds. In model (2), the flywheel directly connected to the CVT, then both the flywheel and engine connected to a transmission system. In this method, the velocity of the flywheel is controlled by the CVT, which supplies the required speed for the coupling with the combustion engine. In this method, the velocity of the flywheel depends on the speed of the combustion engine, and since the combustion engine should be work at the optimum engine speed by reducing power and engine speed, the rest of the power is received through a flywheel.

The design speed of the flywheel independently of the engine is controlled by a CVT, this increases the cost of power transmission systems. In the model (4) flywheel systems and CVT are connected to the end of powertrain, in this way, the combustion engine is controlled by its gearbox, and the flywheel is controlled by its CVT, and the power of the vehicle is ultimately provided with the power coupling of these two generators. One of the advantages of this system is the reduction of the power transmission path in engaging with gear, which itself reduces friction and increases efficiency. Another advantage of this system is the short flywheel path to the car's wheels because this short path increases the energy absorption efficiency of the brake. In this method, due to the control of the speed of two power generators separately, there is no need for the advanced control system, and the control of the system is more comfortable than the rest of the systems. Moreover, this model provides possible regenerative braking. In this paper, applying the regenerative braking is desired, so model (4) is reasonable and chose.

## 2.4 Controller

This study aims to use the flywheel to accelerate the bus. According to the previous studies, the highest consumption of a city bus takes place at the time of acceleration that by eliminating fuel consumption during this time, the reduction of fuel consumption is significant. Hence, in this paper, the flywheel is designed in such a way that it is controlled in a vehicle acceleration time when the combustion engine is out of the circuit, and the flywheel enters the bus powertrain system. As long as the flywheel can supply the power required by the vehicle, the combustion engine will not enter the system. If the capacity of the flywheel is less than the required power of the bus, the combustion engine enters the system and provides the power required by the bus. During braking, the flywheel engages with the brake system, and the mechanical brake is going out of the system, and the flywheel absorbs the car brake energy. As long as the flywheel can supply brake acceleration, which is required for the bus, the mechanical brake system does not enter the system (to the acceleration of  $0.3 \text{ m/s}^2$ ). If the bus has a severe braking acceleration, the mechanical brake will be inserted, and the bus will be braked.

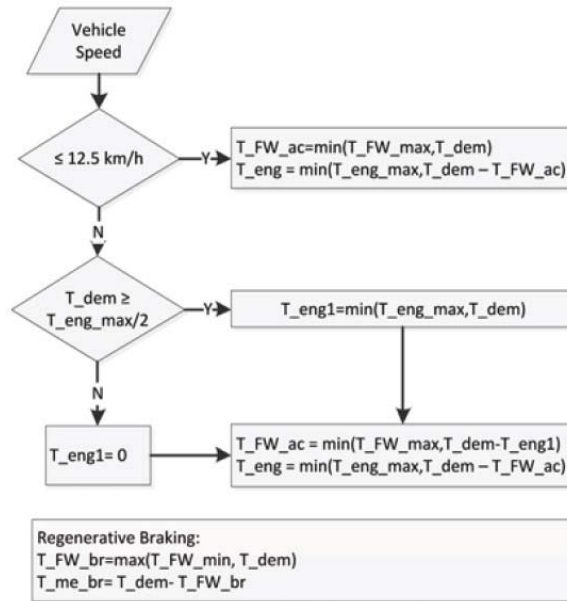
The controller needs to be created concerning illustration, a straightforward lead built controller, which demonstrations clinched alongside three modes (Safaei et al., 2012). To these modes, those fundamental Hybrid controllers determine those acceleration flywheel torques ( $T_{FW\_ac}$ ), those regenerative flywheel torques ( $T_{FW\_br}$ ), the motor torque ( $T_{eng}$ ) and the mechanical brake torque ( $T_{brake}$ ). These amounts need aid ascertained for respect to the highest flywheel torque ( $T_{FW\_max}$ ), the highest motor torque ( $T_{eng\_max}$ ), and the demanded accelerating torques ( $T_{dem}$ ). Those tenets are created dependent upon those states over which that internal combustion engine (ICE) works proficiently. Moreover, the most amount of kinetic energy can be regenerated, and the demanded flywheel torque can be produced. That point the ICE and mechanical brake. Torque commands are determined toward subtracting those FW torques starting with those driver interest.

Figure 4 indicates the flowchart of the rule-based control method outlined to the hybrid flywheel transport powertrain.

The controller has been developed as a simple rule-based controller, which acts in three modes (Zahabi et al., 2014). In these modes, the central hybrid controller determines the acceleration flywheel torque ( $T_{FW\_ac}$ ), the regenerative flywheel torque ( $T_{FW\_br}$ ), the engine torque ( $T_{eng}$ ) and the mechanical brake torque ( $T_{brake}$ ).

These quantities are calculated concerning the maximum flywheel torque ( $T_{FW\_max}$ ), the maximum engine torque ( $T_{eng\_max}$ ), and command for the braking and accelerating torques ( $T_{dem}$ ).

**Figure 4** Flowchart of the rule-based control strategy designed for the hybrid flywheel bus powertrain



Source: Esfahanian et al. (2014)

The rules are generated based on the conditions in which the ICE operates efficiently, and the most amount of kinetic energy can be regenerated. By using this control strategy, the FW torque command is generated. Then the ICE and mechanical brake torque commands are derived by subtracting the FW torque from the driver demand. Figure 4 shows a flowchart of the rule-based control strategy designed for the hybrid flywheel bus powertrain.

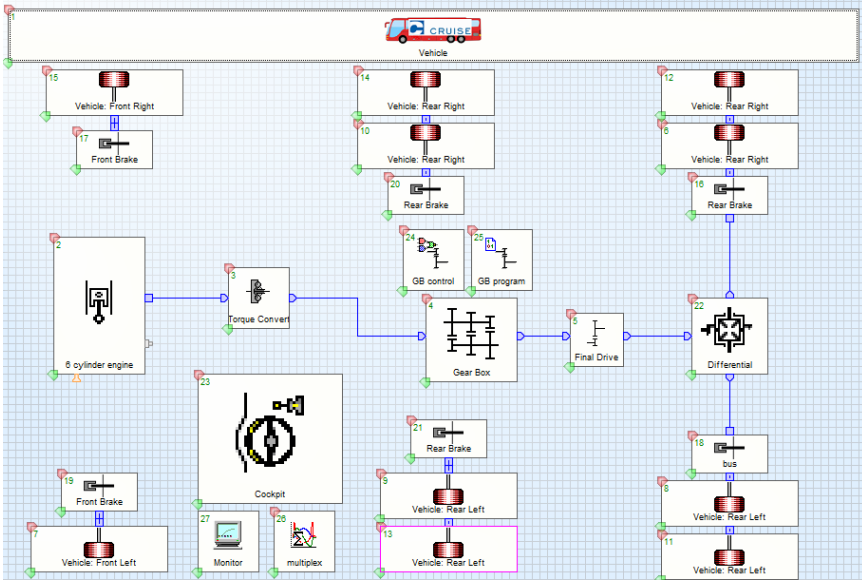
## 2.5 AVL CRUISE software modelling

The ADVISOR (advanced vehicle simulator) may be compelling toward rapidly contrasting vehicle configurations, yet is not expected. However, for modern hybrid powertrain configuration, such as a hybrid flywheel, the ADVISOR needs some modifications. Those vehicle reenactment products chose to ponder. Extent extender motor might have been AVL CRUISE. AVL CRUISE Might, a chance to be arranged, should work (Wahono et al., 2015). By using forward-facing, retrograde facing, or consolidation of the two methodologies, which determined according to the need of the designer. The system AVL mind-boggling may be exceeding. Specialised, it may be guided by the result of assignments in the field for the car industry. Those mind-boggling may be fit should check independently every last bit knots and units of the car and the control framework. Need full library from claiming knots of the auto for modelling: car

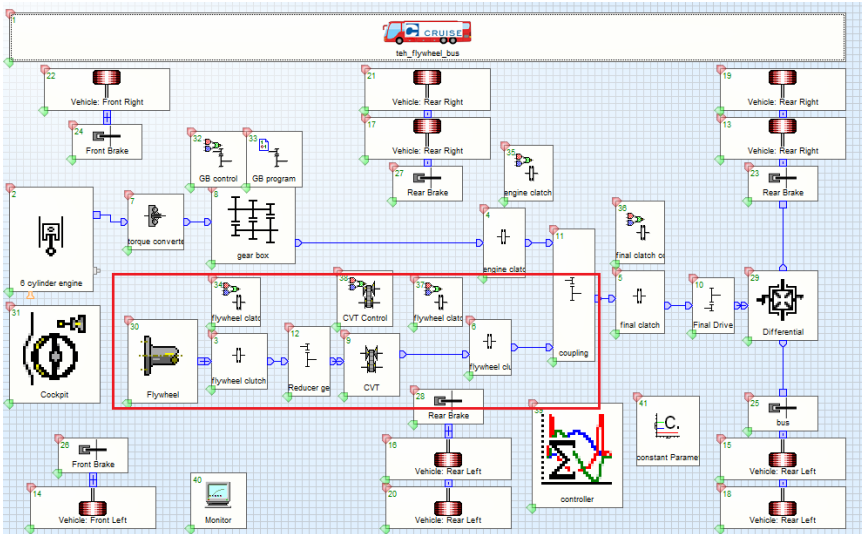


also trucks, buses, motorcycles, mixture what is more electric autos (Ilmbetov et al., 2015). With recreate the bus, the information about each framework segments must provide information under those programming. The re-enactment model, all aspects of transport powertrain for their association, must provide a chance to be. Precisely design of the software, for case controller, may be used to control programmed transmission. Figure 5 hint at traditional transport in the AVL journey workspace.

**Figure 5** Conventional bus modelling in AVL CRUISE (see online version for colours)



**Figure 6** Flywheel hybrid bus powertrain model in AVL CRUISE software workspace (see online version for colours)



To simulate the hybrid components that need to hybrid buses will be added to the conventional bus model. After wrapping flywheel blocks in the model, the block immediately connects to the clutch to disconnect between the flywheel and the transmission. The clutch is connected to a CVT system. The task of CVT is determined at a proper speed and torque, by selecting the appropriate ratio. The selection of appropriate gear ratio is doing by the CVT controller, which has a particular block. On the other side, the ICE is connected to an automatic transmission. Finally, the two parallel power transmission system are coupled together and make the hybrid bus. Figure 6 shows the flywheel hybrid bus powertrain model in AVL CRUISE software workspace.

The information on the bus that input the software model is shown in Table 1.

**Table 1** Bus characteristics

<i>Parameter</i>	<i>Value</i>	
Constructive dimensions	Length	17,800 mm
	Width	2,500 mm
	Height	3,100 mm
	Passenger transport capacity	130
	Tire sizes	256/70R19.5
	Weight without passengers	18,000 kg
Dynamic performances	Diesel tank capacity	300 l
	Maximum velocity	80 km/h
Engine characteristics	Gyration radius	12
	Maximum power	221 kW @2,100 rpm
	Maximum torque	1,250 Nm @1,200 rpm

*Source:* City Buses (2016)

### 3 Flywheel sizing

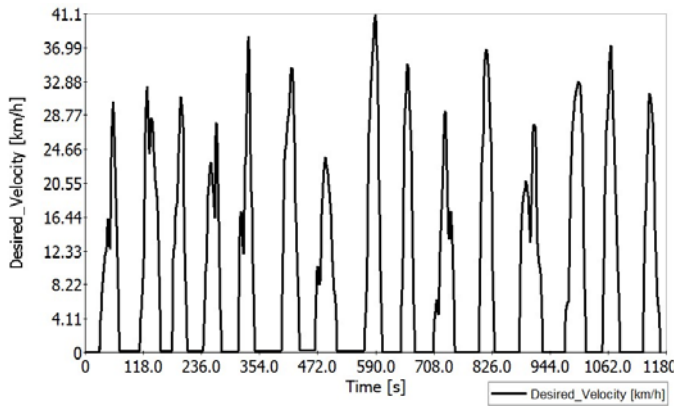
#### 3.1 Driving cycle

The driving cycle is such a vital parameter to simulate vehicle fuel consumption because the movement behaviour of a vehicle determines by that (Günther et al., 2017). In this article, the rapid bus-driving cycle and Tehran City bus driving cycle are using to compare vehicle behavior. The rapid bus-driving cycle has a disciplined movement because the stations are located at precise distance and bus paths the way by a specific range of speed and acceleration. The rapid bus-driving cycle is visible in Figure 7.

In this driving cycle, the maximum speed 40.77 km/h, the total distance 2,931.36 m, the whole movement time 1,167 second, the average positive acceleration 0.413 m/s<sup>2</sup>, and the average negative acceleration is -0.584 m/s<sup>2</sup>. In Figure 8, Tehran City bus driving cycle is visible.

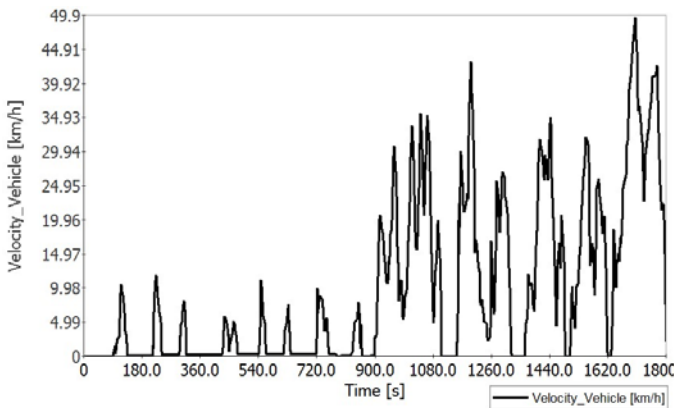
In this driving cycle, the maximum speed 50.1 km/h, the total distance 4,871.36 m, the whole movement time 1,800 second, the average positive acceleration 0.34 m/s<sup>2</sup>, and the average negative acceleration is -0.52 m/s<sup>2</sup>.

**Figure 7** BRT driving cycle



*Source:* Lai et al. (2013)

**Figure 8** Tehran City bus driving cycle



*Source:* Masih-Tehrani et al. (2012)

### 3.2 Initial design

The driving torque demand decision refers to the relationship between the accelerator pedal opening and driving torque demand, which directly affects the vehicle’s dynamics property, comfort, and fuel economy (Shen et al., 2017). One of the most important and essential parameters necessary to select flywheel power. Table 2 identifies the power required to move at maximum speed.

According to the proposed scheme for hybrid buses, one of the most critical parameters in the selection of the flywheel is the power that is required to accelerate the vehicle, according to work done in the field. Table 3 shows the power of flywheel, which is proposed to accelerate the bus.

**Table 2** Bus power required

<i>Operating conditions</i>	<i>Required power</i>
Speed of 80 km/h slope pf 0%	65
Speed of 40 km/h slope pf 7%	155
Speed of 35 km/h slope pf 10%	184
Speed of 20 km/h slope pf 14%	142

*Source:* Masih-Tehrani et al. (2012)

**Table 3** The power required for bus acceleration

<i>Operating conditions</i>	<i>Power demands</i>
0–25 in 10 seconds	89 kW
25–35 in 5 seconds	101 kW
35–40 in 5 seconds	59 kW
40–44 in 5 seconds	53 kW
0–60 in 60 seconds	100 kW

*Source:* Wahono et al. (2015)

According to the design method in this paper, the purpose of the bus acceleration is using a single flywheel without the use of a combustion engine, that is, it means until the flywheel has the ability to move and accelerate the bus and it is itself can provide the bus power and the need for a bus to power more than the power stored in the flywheel of the combustion engine helps the flywheel to accelerate. Therefore, we will consider the power which is needed for the flywheel in the worst case and the maximum power required to apply the bus acceleration. Then to achieve bus speed from 0 to 60 km/h, the flywheel must have a minimum power of 100 kW. So about power for hybrid buses is achieved 100 kW. Whit considers safety factor 1.2; the flywheel power is selected 120 kW. The other important parameter to specify the flywheel energy storage capacity, different values for flywheel energy storage capacity, is considered, and according to the simulation, the reduction in fuel consumption achieved by the flywheel be calculated. By calculate the initial cost of the flywheel and decrease fuel consumption per year, the return on investment was calculated, and optimal flywheel energy storage capacity is selected. The fundamental properties of the flywheel can be seen in Table 4.

**Table 4** Flywheel specification

<i>Parameter</i>	<i>Dimension</i>	<i>Value</i>
Maximum power	kW	120
Maximum speed	RPM	37,000
Minimum speed	RPM	15,000
Efficiency	%	92
Moment of inertia	kg.m <sup>2</sup>	0.306
Power density	kW/kg	2.2
Total weight	kg	55
Life span	Year	20

*Source:* Ilimbetov et al. (2015)

### 3.3 Flywheel cost

According to Torotrak company statements, the flywheel costs per year in 2015 is USD500 per kWh, If the value of this cost in the year 2011, USD1,000 to 2,000 per kWh flywheel energy. In this article is intended flywheel approximates cost USD1,000 per kWh. By turning the unit, the flywheel price is computing USD277 per MJ, simplifying, flywheel cost is considered USD300 per MJ. The cost of CVT and other equipment use in the flywheel system be considered USD800. So the flywheel system cost in terms of energy storage capacity in Table 5 is visible (Doucette and McCulloch, 2011).

**Table 5** Price flywheel system in terms of energy capacity.

<i>Flywheel energy storage capacity (MJ)</i>	<i>Total price flywheel system (USD)</i>
0.4	920
0.8	1,040
1.2	1,160
1.4	1,220
1.8	1,340
2.2	1,460
2.6	1,580
3	1,700

### 3.4 Optimising energy capacity

The following calculations are assuming that the bus cycles measured in each of the 200 km travel. Gasoline price is considered USD0.125 per litre. Since fuel consumption of conventional buss in the BRT driving cycle is achieved 57.68 lit/100km, so the bus fuel consumption is 115.36 litres a day in this driving cycle. Table 6 shows the reducing of fuel consumption and cost savings.

**Table 6** Energy capacity in terms of reducing the cost

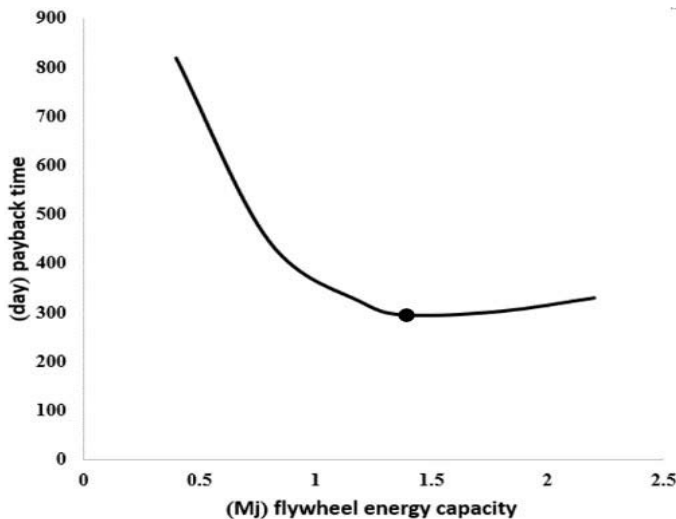
<i>Flywheel energy storage capacity (MJ)</i>	<i>Reducing fuel consumption</i>	
	<i>(%)</i>	<i>(Lit per day)</i>
0.4	7.8	9
0.8	16.18	18.67
1.2	25.03	28.87
1.4	28.61	33
1.8	30.62	35.32
2.2	30.62	35.32

The fuel consumption increases with increased capacity flywheel energy; on the other hand, increases the initial cost is increased capacity flywheel energy. Table 7 displays the payback period.

Figure 9 is displayed flywheel energy capacity in terms of the payback period.

**Table 7** Energy capacity in terms of the payback period

<i>Flywheel energy storage capacity (MJ)</i>	<i>Initial cost (USD)</i>	<i>Payback period</i>
0.4	920	2 years 2 months 28 days
0.8	1,040	1 year 2 months 21 days
1.2	1,160	10 months 22 days
1.4	1,220	9 months 27 days
1.8	1,340	10 months 4 days
2.2	1,460	11 months 1 day

**Figure 9** Energy capacity in terms of the payback period

Increased flywheel energy capacity to 1.4 MJ either reduces fuel consumption, nor reduces payback period initial cost time, after 1.4 MJ of flywheel energy capacity increases payback period, so the flywheel energy capacity to reduce fuel consumption and the payback period is 1.4 MJ. For this driving cycle, the payback period is nine months, 27 days.

## 4 Results

The hybrid flywheel bus with optimal energy storage capacity is simulated in various working situations. The results of AVL CRUISE use to evaluate flywheel bus performance. Figure 10 shows the cycle travel error for the flywheel bus. In this travel cycle, RMSE is 0.242, and the maximum error is 4.2 km/h, and also as it is clear RMSE cycling error is negligible, and bus cycling is acceptable with using the flywheel.

Figure 11, the flywheel speed (energy level) and the vehicle velocity in a driving cycle are shown, which are extracted from the AVL CRUISE software. As a software constraint, the energy level of flywheel is assumed equal both at the beginning and the end of driving cycle. Therefore, the energy recovery rate can be compared in different

design cases. The results show that due to the acceleration and braking capabilities, flywheel has an excellent performance.

**Figure 10** Cycle travel error

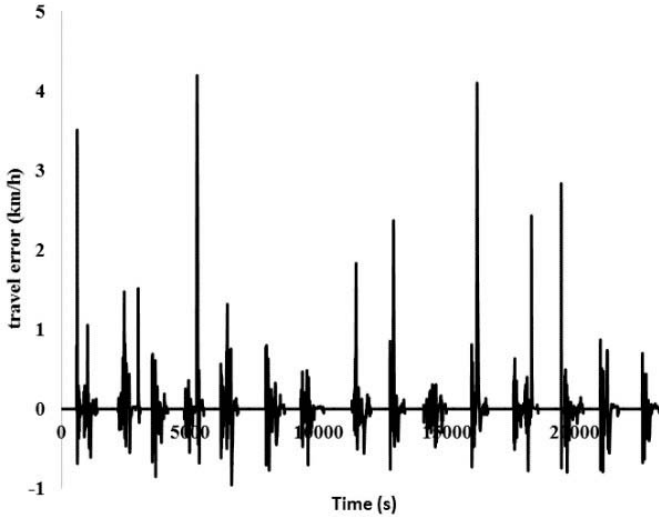
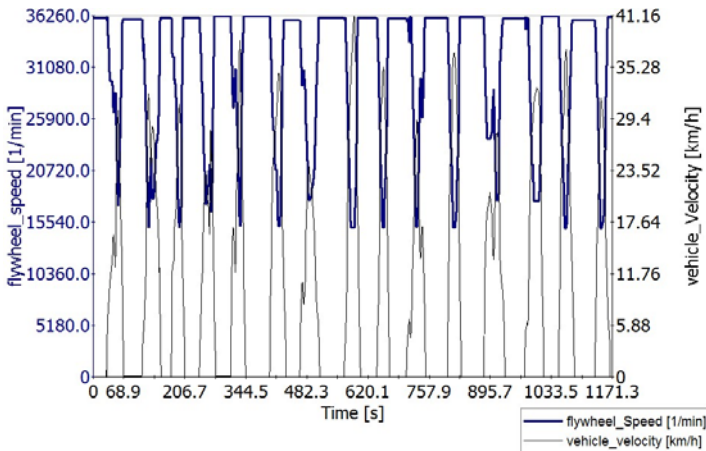


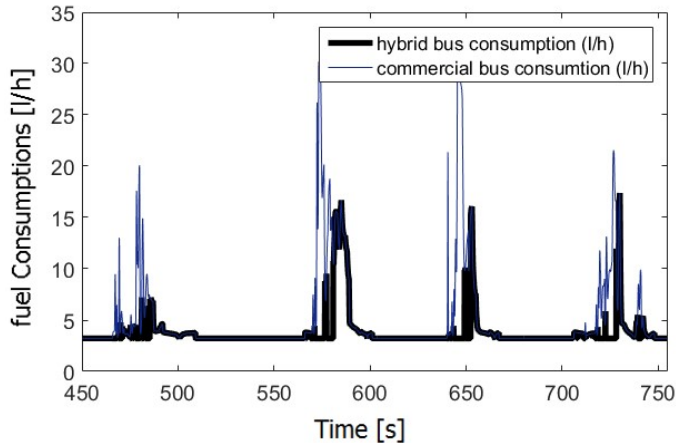
Figure 12 compares the fuel consumptions of flywheel and commercial buses. As shown in this figure, fuel consumption is very high during acceleration in a conventional bus; while in the hybrid flywheel bus, it is moderated very well. For better comparison this figure shows the only range of 450 to 750 seconds of the cycle.

**Figure 11** Flywheel speed and vehicle velocity in the driving cycle (see online version for colours)



To better determine the reason of chosen flywheel bus to use in BRT in this article, compare flywheel bus by 1.4 MJ energy in two cycles, Tehran City bus cycle, and BRT cycle. Table 8 compare the percent fuel consumption reduction and a payback period of two-cycle, which said.

**Figure 12** Fuel consumption comparison between the flywheel and conventional city buses in a period of driving cycle (see online version for colours)



**Table 8** Compare fuel consumption and payback period

<i>Cycle</i>	<i>Reduction of fuel consumption (%)</i>	<i>Reduction of fuel consumption per day (lit)</i>	<i>Payback period</i>
Tehran City bus	18.5	19.17	1 year 4 months 24 days
Bus rapid transit	28.61	33	9 months 27 days

According to Table 8, the reduction of fuel consumption of hybrid flywheel bus in BRT driving cycle is 10.11% more than the reduction of fuel consumption in the Tehran City bus. If we want to choose the best flywheel energy capacity for the reduction of fuel consumption at the highest level will be achieved 2.2 MJ, but if we choose this energy capacity for the flywheel to use in Tehran City bus city also payback period will be lower than BRT cycle. The payback period of the flywheel by 2.2 MJ for Tehran City bus will achieve in one year four months seven days. Table 9 shows the comparison of RMSE and maximum error between conventional buses and flywheel bus. As shown, these parameters have a low variations.

**Table 9** Compare of RMSE and maximum error

<i>Conventional buses bus</i>		<i>Hybrid bus</i>	
<i>RMSE</i>	<i>Max error</i>	<i>RMSE</i>	<i>Max error</i>
0.235	4.051	0.242	4.198

Vehicle acceleration and grade-ability performance for both conventional and hybrid buses are compared and shown in Table 10.

**Table 10** Compare the performance of the bus

<i>Bus type</i>	<i>Accelerating time 0–40 km/h (sec)</i>	<i>Max grade-ability (%)</i>
Conventional bus	14.37	26.11
Flywheel hybrid bus	12.18	26.02



The flywheel hybrid bus has a significant better acceleration than the commercial bus. The flywheel pack has no effect on the climbing performance of the bus. Therefore, a little difference in grade-ability performance (the third column of Table 10) is related to the additive weight of the flywheel pack on the hybrid bus. The flywheel system weight has been considering 150 kg, in this study.

## 5 Conclusions

In this paper, the design procedure for the proposed flywheel mechanical powertrain was explained. The reduction in fuel consumption and the initial cost required to install the flywheel was calculated to find the best flywheel energy storage capacity for the BRT driving cycle. After finding the best flywheel energy capacity, we compared the performance of hybrid flywheel bus and conventional bus in two driving cycles contains BRT driving cycle and Tehran City bus driving cycle.

Simulation results show a considerable decrease in fuel consumption as well as improvements in vehicle performance compared with a conventional vehicle. In comparison with the Tehran City bus driving cycle, flywheel hybrid bus which uses in the BRT cycle, have an excellent efficiency. The optimal flywheel energy storage capacity is 1.4 MJ to use in the BRT cycle. Furthermore, the payback period is nine months 27 days besides that reduction of fuel consumption is 28.61%. It is highly recommended for those who want to study in this context as long as they use a sophisticated controller (such as developed in Esfandyari et al., 2019; Masih-Tehrani et al., 2019) to control the flywheel and engine simultaneously, in order to get the engine (Salavati-Zadeh et al., 2016; Ghavami et al., 2018) working conditions at optimal points in all the time of cycle travel. Another suggestion is that the combination flywheel hybrid with other available hybrids such as supercapacitor and battery (Masih-Tehrani and Dahmardeh, 2018; Rahimirad et al., 2019) to improve storage performance, in order to use it for working in extra time.

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